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$\psi(3770)$ and B meson exclusive decay B $\rightarrow \psi(3770)$K in QCD factorization

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Abstract. Belle has observed surprisingly copious production of $\psi(3770)$ in the B meson decay $B \rightarrow \psi(3770)K$, whose rate is comparable to that of $B \rightarrow \psi(3686)K$. We study this puzzling process in the QCD factorization approach with the effect of the S-D mixing considered. We find that the soft scattering effects in the spectator interactions play an essential role. With a proper parametrization for the higher-twist soft end-point singularities associated with kaon, and with the S-D mixing angle $\theta = -12°$, the calculated decay rates can be close to the data. Implications of these soft spectator effects to other charmonium production in B exclusive decays are also emphasized.

PACS. 13.25.Hw Decays of bottom mesons – 12.38.Bx Perturbative calculations – 14.40.Gx Mesons with $S = C = B = 0$, mass > 2.5 GeV (including quarkonia)

1 Introduction

The $\psi(3770)$ is the lowest-lying charmonium state above the open-charm $D\bar{D}$ threshold. It is expected to be predominantly the 1^3D_1 charmonium state with a small admixture of the 2^3S_1 component. The $\psi(3770)$ is of great interest in recent studies of charmonium physics. There are a number of new measurements and related theoretical issues about the $\psi(3770)$, *e.g.* the non–D-$\bar{D}$ decays including charmonium transitions and decays to light hadrons [1–3] (see also [4]), the radiative transitions to the P-wave charmonia [5], the S-D mixing, and the discussions about the well-known $\rho\pi$ puzzle in J/ψ and $\psi(3686)$ decays (see, *e.g.*, [6,7]).

In this paper, we will focus on another interesting issue about the $\psi(3770)$. That is the $\psi(3770)$ production in the B meson exclusive decay $B \rightarrow \psi(3770)K$, whose rate is found by Belle to be surprisingly large [8], even comparable to that of $B \rightarrow \psi(3686)K$, and it might seemingly indicate that this result suggests a large amount of S-D mixing in the $\psi(3770)$ [8]. But, this apparently needs a careful examination.

It is generally believed that if the virtual charmed-meson pair components are neglected the two states $\psi(3686)$ and $\psi(3770)$ can be approximately expressed as

$$\begin{aligned} |\psi'\rangle &\equiv |\psi(3686)\rangle = \cos\theta\,\big|c\bar{c}\big(2^3S_1\big)\big\rangle + \sin\theta\,\big|c\bar{c}\big(1^3D_1\big)\big\rangle, \\ |\psi''\rangle &\equiv |\psi(3770)\rangle = \cos\theta\,\big|c\bar{c}\big(1^3D_1\big)\big\rangle - \sin\theta\,\big|c\bar{c}\big(2^3S_1\big)\big\rangle. \end{aligned} \quad (1)$$

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The S-D mixing angle has been estimated by using the ratio of the leptonic decay widths [9] of $\psi(3686)$ and $\psi(3770)$. Nonrelativistic potential model calculations give two solutions: $\theta \approx -10°$ to $-13°$ or $\theta \approx +30°$ to $+26°$ [6,4,10]. The small mixing angle is compatible with the results obtained in models with coupled-channel effects [11,12] and is favored by the $E1$ transition $\psi' \rightarrow \gamma\chi_{cJ}$ data also [10].

The Belle Collaboration [8] has observed $\psi(3770)$ in the B meson decay $B^+ \rightarrow \psi(3770)K^+$ with a branching ratio,

$$\mathrm{Br}\left(B^+ \rightarrow \psi''K^+\right) = (0.48 \pm 0.11 \pm 0.07) \times 10^{-3}, \quad (2)$$

which is comparable to that of $\psi(3686)$ [9],

$$\mathrm{Br}\left(B^+ \rightarrow \psi'K^+\right) = (0.66 \pm 0.06) \times 10^{-3}. \quad (3)$$

This is quite surprising, since conventionally the $\psi(3770)$ and $\psi(3686)$ are regarded as predominantly the 1^3D_1 and 2^3S_1 $c\bar{c}$ states, respectively, and the coupling of 1^3D_1 to the $c\bar{c}$ vector current in the weak-decay effective Hamiltonian is much weaker than that of 2^3S_1 in the naive factorization approach [13]. If this experimental result is really due to a large S-D mixing, as suggested in [8], then it is found in ref. [14] that an unexpectedly large S-D mixing angle $\theta = \pm 40°$ would be required by fitting the observed ratio of $B \rightarrow \psi(3770)K$ to $B \rightarrow \psi(3686)K$ decay rates, when the D-wave contribution is neglected. This is in serious contradiction with all other experimental and theoretical studies, and, in particular, with the newly measured

$E1$ transition rates for $\psi(3770) \to \gamma\chi_{cJ}$ $(J = 0, 1, 2)$, for which the CLEO results [5] are $172\pm30, 70\pm17, < 21$ KeV, respectively, for $J = 0, 1, 2$ whereas the corresponding calculations are 386, 0.32, 66 KeV for $\theta = -40°$ and 52, 203, 28 KeV for $\theta = +40°$ [14]. So, based on the naive factorization, the use of large S-D mixing to explain the Belle data for $B \to \psi(3770)K$ should be ruled out. The next question is, can we explain the Belle data by considering the nonfactorizable contributions to these decay rates?

In the following, we will study this problem in the QCD factorization approach [15–17] including nonfactorizable contributions. We will first give the decay rate of $B \to \psi(3770)K$ based on the assumption that $\psi(3770)$ is a pure D-wave charmonium state. Then we take the S-D mixing into account. Finally, we will consider the higher-twist effects.

2 B → ψ(3770)K decay in QCD factorization

The effective Hamiltonian for this decay mode is written as [18]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\left(V_{cb}V_{cs}^*(C_1\mathcal{O}_1 + C_2\mathcal{O}_2) - V_{tb}V_{ts}^*\sum_{i=3}^{10} C_i\mathcal{O}_i\right). \quad (4)$$

Here C_i's are the Wilson coefficients which can be evaluated by the renormalization group approach [18] and the results at $\mu = 4.4$ GeV are listed in table 1. The relevant operators $\mathcal{O}_i$ in H_{eff} are given by

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot (\bar{c}_\beta c_\alpha)_{V-A},\\
\mathcal{O}_2 &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot (\bar{c}_\beta c_\beta)_{V-A},\\
\mathcal{O}_{3(5)} &= (\bar{s}_\alpha b_\alpha)_{V-A} \cdot \sum_q (\bar{q}_\beta q_\beta)_{V-A(V+A)},\\
\mathcal{O}_{4(6)} &= (\bar{s}_\alpha b_\beta)_{V-A} \cdot \sum_q (\bar{q}_\beta q_\alpha)_{V-A(V+A)},\\
\mathcal{O}_{7(9)} &= \frac{3}{2}(\bar{s}_\alpha b_\alpha)_{V-A} \cdot \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A(V-A)},\\
\mathcal{O}_{8(10)} &= \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \cdot \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A(V-A)}.
\end{aligned} \quad (5)$$

We treat the charmonium as a color-singlet nonrelativistic $c\bar{c}$ bound state. Let p_μ be the total 4-momentum of the charmonium and $2q_\mu$ be the relative 4-momentum between c and $\bar{c}$ quarks. For D-wave charmonium, because the wave function and its first derivative at the origin vanish, $\mathcal{R}_1(0) = 0$, $\mathcal{R}_1'(0) = 0$, which correspond to the zeroth and the first order in q, we must expand the amplitude to second order in q. Thus, we have (see, *e.g.*, [19])

$$\begin{aligned}
&\mathcal{M}\left(B \to {}^3D_1(c\bar{c})\right) = \frac{1}{2}\sum_{L_z,S_z}\langle 2L_z; 1S_z|1J_z\rangle\\
&\times \int \frac{\mathrm{d}^4q}{(2\pi)^3} q_\alpha q_\beta \delta\left(q^0 - \frac{|\vec{q}|^2}{M}\right)\psi_{2M}^*(q)\\
&\times \mathrm{Tr}\left[\mathcal{O}^{\alpha\beta}(0)P_{1S_z}(p,0) + \mathcal{O}^\alpha(0)P_{1S_z}^\beta(p,0)\right.\\
&\left.+\mathcal{O}^\beta(0)P_{1S_z}^\alpha(p,0) + \mathcal{O}(0)P_{1S_z}^{\alpha\beta}(p,0)\right],
\end{aligned} \quad (6)$$

Table 1. Leading-order (LO) and Next–to–leading-order (NLO) Wilson coefficients in the NDR scheme (see ref. [18]) with $\mu = 4.4$ GeV and $\Lambda^{(5)}_{\overline{\text{MS}}} = 225$ MeV.

	C_1	C_2	C_3	C_4	C_5	C_6
LO	1.144	−0.308	0.014	−0.030	0.009	−0.038
NDR	1.082	−0.185	0.014	−0.035	0.009	−0.041

where $\mathcal{O}(q)$ represents the rest of the decay matrix element. The spin-triplet projection operators $P_{1S_z}(p,q)$ is constructed in terms of quark and anti-quark spinors as

$$P_{1S_z}(p,q) = \sqrt{\frac{3}{m}}\sum_{s_1,s_2} v\left(\frac{p}{2} - q, s_2\right)\bar{u}\left(\frac{p}{2} + q, s_1\right)\langle s_1; s_2|1S_z\rangle, \quad (7)$$

and

$$\begin{aligned}
&\mathcal{O}^\alpha(0) = \frac{\partial\mathcal{O}(q)}{\partial q_\alpha}|_{q=0}, \qquad \mathcal{O}^{\alpha\beta}(0) = \frac{\partial^2\mathcal{O}(q)}{\partial q_\alpha \partial q_\beta}|_{q=0},\\
&P_{1S_z}^\alpha(p,0) = \frac{\partial P_{1S_z}(p,q)}{\partial q_\alpha}|_{q=0},\\
&P_{1S_z}^{\alpha\beta}(p,0) = \frac{\partial^2 P_{1S_z}(p,q)}{\partial q_\alpha \partial q_\beta}|_{q=0}.
\end{aligned} \quad (8)$$

After q^0 is integrated out, the integral in eq. (6) is proportional to the second derivative of the D-wave wave function at the origin by

$$\int \frac{\mathrm{d}^3q}{(2\pi)^3} q^\alpha q^\beta \psi_{2m}^*(q) = e_m^{*\alpha\beta}\sqrt{\frac{15}{8\pi}}\mathcal{R}_D''(0), \quad (9)$$

where $e_m^{\alpha\beta}$ is the polarization tensor of an angular momentum-2 system and the value of $\mathcal{R}_D''(0)$ for charmonia can be found in, *e.g.*, ref. [20].

The spin projection operators $P_{1S_z}(p,0)$, $P_{1S_z}^\alpha(p,0)$ and $P_{1S_z}^{\alpha\beta}(p,0)$ can be written as [19]

$$\begin{aligned}
&P_{1S_z}(p,0) = \sqrt{\frac{3}{4M}}\not{\epsilon}^*(S_z)(\not{p} + M), \quad (10)\\
&P_{1S_z}^\alpha(p,0) = \sqrt{\frac{3}{4M^3}}[\not{\epsilon}^*(S_z)(\not{p}+M)\gamma^\alpha + \gamma^\alpha\not{\epsilon}^*(S_z)(\not{p}+M)],\\
&P_{1S_z}^{\alpha\beta}(p,0) = \sqrt{\frac{3}{4M^5}}\left[\gamma^\beta\not{\epsilon}^*(S_z)(\not{p} + M)\gamma^\alpha\right.\\
&\qquad\left.+\gamma^\alpha\not{\epsilon}^*(S_z)(\not{p} + M)\gamma^\beta\right], \quad (11)
\end{aligned}$$

where we have made use of the nonrelativistic approximation for the charmonium mass $M \simeq 2m$. Here m is the charmed-quark mass.

As for the light meson kaon, we describe it relativistically by light-cone distribution amplitudes (LCDAs) [17] up to the twist-3 level:

$$\langle K(p)|\bar{s}_\beta(z_2)\, d_\alpha(z_1)|0\rangle = \frac{if_K}{4}\int_0^1 \mathrm{d}x e^{i(y\,p\cdot z_2+\bar{y}\,p\cdot z_1)} \times\bigg\{\not p\,\gamma_5\,\phi_K(y) - \mu_K\gamma_5\bigg(\phi_K^p(y) -\sigma_{\mu\nu}\,p^\mu\big(z_2-z_1\big)^\nu\,\frac{\phi_K^\sigma(y)}{6}\bigg)\bigg\}_{\alpha\beta}, \quad (12)$$

where y and $\bar{y} = 1-y$ are the momentum fractions of the s and $\bar{d}$ quarks inside the K meson, respectively. Here, the chirally enhanced mass scale $\mu_K = {m_K}^2/(m_s(\mu)+m_d(\mu))$ is comparable to m_b, which ensures that the twist-3 spectator interactions are numerically large, though they are suppressed by $1/m_b$. The twist-2 LCDA $\phi_K(y)$ and the twist-3 ones $\phi_K^p(y)$ and $\phi_K^\sigma(y)$ are symmetric under $y \leftrightarrow \bar{y}$ in the limit of $SU(3)$ isospin symmetry. In practice, we choose the asymptotic forms for these LCDAs,

$$\phi_K(y) = \phi_K^\sigma(y) = 6y(1-y), \qquad \phi_K^p(y) = 1. \quad (13)$$

In the naive factorization, we neglect the strong-interaction corrections and the power corrections in $\Lambda_{\mathrm{QCD}}/m_b$. Then, the decay amplitude can be written as

$$i\mathcal{M}_0 = -f_D m_{\psi''}(2p_B\cdot\varepsilon^*)F_1\big(m_{\psi''}^2\big)\frac{G_F}{\sqrt{2}} \times\bigg[V_{cb}V_{cs}^*\bigg(C_2+\frac{C_1}{N_c}\bigg)-V_{tb}V_{ts}^*\bigg(C_3+\frac{C_4}{N_c}+C_5+\frac{C_6}{N_c}\bigg)\bigg], \quad (14)$$

where N_c is the number of colors. We do not include the effects of the electroweak penguin operators since they are numerically small. The form factors for $B \to K$ are given as

$$\langle K(p_K)|\bar{s}\gamma_\mu b|B(p_B)\rangle = \bigg[(p_B+p_K)_\mu - \frac{m_B^2-m_K^2}{p^2}p_\mu\bigg]F_1\big(p^2\big) +\frac{m_B^2-m_K^2}{p^2}p_\mu F_0\big(p^2\big), \quad (15)$$

where $p = p_B - p_K$ is the momentum of ψ'' with $p^2 = m_{\psi''}^2$. The kaon mass will be neglected in the heavy quark limit and we will use the approximate relation $F_0(m_{\psi''}^2)/F_1(m_{\psi''}^2) = 1 - r$ [21,22], where $r = m_{\psi''}^2/m_B^2$, to simplify the amplitude in our calculations.

As we can easily see in eq. (14), this amplitude is unphysical because the Wilson coefficients depend on the renormalization scale μ while the decay constant and the form factors are independent of μ. This is the well-known problem with the naive factorization. However, if we include the order α_s corrections, it turns out that the μ-dependence of the Wilson coefficients is largely cancelled and the overall amplitude is insensitive to the renormalization scale. Taking the nonfactorizable order α_s strong-interaction corrections in fig. 1 into account, the full decay amplitude for $B \to \psi''K$ within the QCD factorization approach is written as

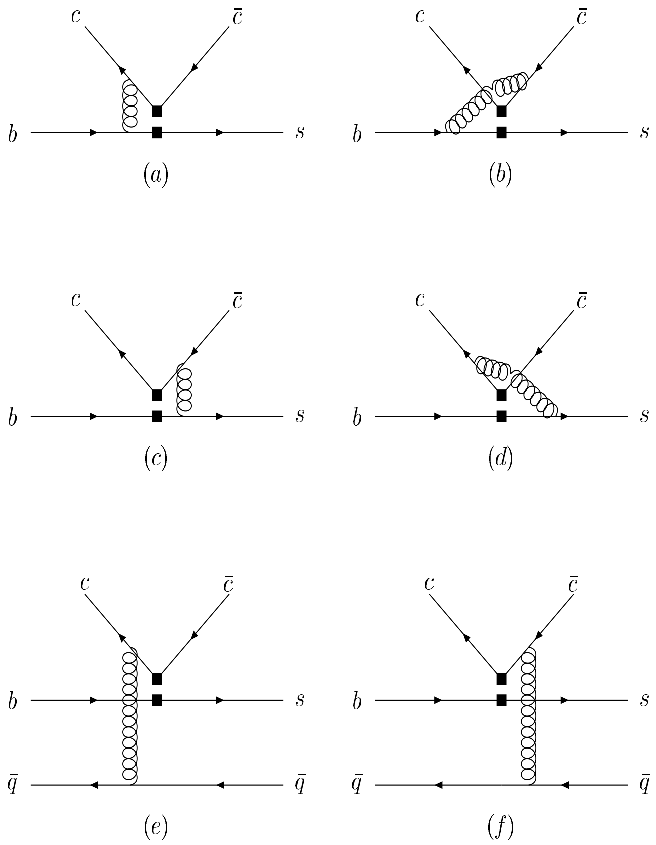

Fig. 1. Feynman diagrams for nonfactorizable corrections to the $B \to \psi''K$ decay.

$$i\mathcal{M} = f_D m_{\psi''}(2p_B\cdot\varepsilon^*)F_1\big(m_{\psi''}^2\big)\frac{G_F}{\sqrt{2}} \times\Big[V_{cb}V_{cs}^*a_2 - V_{tb}V_{ts}^*(a_3+a_5)\Big], \quad (16)$$

where the coefficients a_i $(i = 2,3,5)$ in the naive dimension regularization (NDR) scheme are given by

$$\begin{aligned} a_2 &= -\bigg(C_2+\frac{C_1}{N_c}\bigg) + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_1\bigg(-12\ln\frac{m_b}{\mu}+2+f_I+f_{II}\bigg),\\ a_3 &= -\bigg(C_3+\frac{C_4}{N_c}\bigg) + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_4\bigg(-12\ln\frac{m_b}{\mu}+2+f_I+f_{II}\bigg),\\ a_5 &= -\bigg(C_5+\frac{C_6}{N_c}\bigg) - \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_6\bigg(-12\ln\frac{m_b}{\mu}-10+f_I+f_{II}\bigg). \end{aligned} \quad (17)$$

The function f_I in eq. (17) is calculated from the four vertex diagrams (a, b, c, d) in fig. 1,

$$f_I = \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y$$
$$\times \Bigg[-6\ln\left[\left(x+\frac{y}{2}\right)\left(x+\frac{ry}{2}\right)\frac{y}{2}\left((r-1)x+\frac{ry}{2}\right)\right]$$
$$-\frac{3}{5}(1-r)^2x^2y^2\left(\frac{1}{\left(x+\frac{y}{2}\right)^2\left(x+\frac{ry}{2}\right)^2}+\frac{1}{\left(\frac{y}{2}\left((r-1)x+\frac{ry}{2}\right)\right)^2}\right)$$
$$-2ry(1-y)\left(\frac{1}{\left(x+\frac{y}{2}\right)\left(x+\frac{ry}{2}\right)}+\frac{1}{\frac{y}{2}\left((r-1)x+\frac{ry}{2}\right)}\right)$$
$$-2\left(\frac{(1+r)x-r(2-y)}{x+\frac{ry}{2}}+\frac{(r-1)x-r(2-y)}{(r-1)x+\frac{ry}{2}}\right)$$
$$-2r(1-r)xy^2\left(\frac{1}{\left(x+\frac{y}{2}\right)\left(x+\frac{ry}{2}\right)^2}-\frac{1}{\frac{y}{2}\left((r-1)x+\frac{ry}{2}\right)^2}\right)$$
$$+\frac{2}{5}r(1-r)^2x^2y^2\left(\frac{1}{\left(x+\frac{y}{2}\right)\left(x+\frac{ry}{2}\right)^3}+\frac{1}{\frac{y}{2}\left((r-1)x+\frac{ry}{2}\right)^3}\right)\Bigg], \tag{18}$$

where $r = m_{\psi''}^2/m_B^2$.

The function f_{II} in eq. (17) is calculated from the two spectator interaction diagrams (e,f) in fig. 1 and it is given by

$$f_{II} = \frac{16\pi^2}{N_c}\frac{f_K f_B}{m_B^2 F_1(m_{\psi''}^2)}\int_0^1 \mathrm{d}\xi\frac{\phi_B(\xi)}{\xi}\int_0^1 \mathrm{d}y\phi_K(y)$$
$$\times\left[-\frac{1}{10}\frac{1}{(1-r)(1-y)}-\frac{r}{(1-y)^2(1-r)^2}\right], \tag{19}$$

where ϕ_B is the light-cone wave functions for the B meson. The spectator contribution depends on the wave function ϕ_B through the integral

$$\int_0^1 \mathrm{d}\xi\frac{\phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}. \tag{20}$$

Since $\phi_B(\xi)$ is appreciable only for ξ of order $\Lambda_{\mathrm{QCD}}/m_B$, λ_B is of order Λ_{QCD}. We will follow ref. [17] to choose $\lambda_B \approx 300\,\mathrm{MeV}$ in the numerical calculation.

It is easily seen from (19) that there is logarithmic end-point singularity in the integration over y when $y \to 1$. It breaks down the factorization even at the leading-twist level. It implicates that the soft mechanisms may be important to this decay mode. To estimate these soft effects, we simply parameterize the end-point singularity as

$$X \equiv \int_0^1 \frac{\mathrm{d}y}{y} = \ln\left(\frac{m_B}{\Lambda_h}\right), \tag{21}$$

where $\Lambda_h \sim 500\,\mathrm{MeV}$ is the typical momentum scale associated with the light quark in the B meson. Furthermore, since the virtuality of the gluon exchanged between the spectator quark and the charm (or anti-charm) quark is $\Lambda_h m_b$, we should multiply a factor $\alpha_s(\sqrt{\Lambda_h m_b})C_i(\sqrt{\Lambda_h m_b})/(\alpha_s(\mu)C_i(\mu))$ to f_{II} in eq. (17), where $\mu \sim m_b$ is the scale at which we evaluate those vertex corrections.

The decay constant f_D is calculated through the potential models

$$f_D = \frac{10\sqrt{3}}{\sqrt{2\pi m_{\psi''}}}\frac{R_D''(0)}{m_{\psi''}^2}. \tag{22}$$

For numerical analysis, we choose $F_1(m_{\psi''}^2) = 0.97$ [23] and use the following input parameters:

$$m_b = 4.8\,\mathrm{GeV}, \quad m_B = 5.28\,\mathrm{GeV}, \quad m_{\psi''} = 3.77\,\mathrm{GeV},$$
$$f_B = 216\,\mathrm{MeV}\ [24], \quad f_K = 160\,\mathrm{MeV}. \tag{23}$$

Then, we get the branching ratio: $\mathrm{Br}(B \to \psi''K) = 1.13\times10^{-5}$. The theoretical calculation is about 40 times lower than the experimental data (2).

3 B → ψ′K decay

The calculation of the branching ratio for the $B \to \psi'K$ decay is similar to that for $B \to \psi''K$. If one treats ψ' as a pure $2S$-state, the only modification needed to do is to expand the decay amplitudes to zeroth order in the q-expansion. Thus, the amplitudes will be proportional to the S-wave wave function at the origin through the integration

$$\int \frac{\mathrm{d}^3q}{(2\pi)^3}\psi_{2S}^*(q) = \sqrt{\frac{1}{4\pi}}\mathcal{R}_{2S}(0). \tag{24}$$

The full decay amplitude for $B \to \psi'K$ within the QCD factorization approach is written as

$$i\mathcal{M}' = \sqrt{\frac{3}{\pi m_{\psi'}}}R_{2S}(0)m_{\psi'}(2p_B\cdot\varepsilon^*)F_1\left(m_{\psi'}^2\right)\frac{G_F}{\sqrt{2}}$$
$$\times\left[V_{cb}V_{cs}^*a_2' - V_{tb}V_{ts}^*\left(a_3'+a_5'\right)\right], \tag{25}$$

where the coefficients a_i' ($i = 2,3,5$) in the naive dimension regularization (NDR) scheme are given by

$$a_2' = \left(C_2+\frac{C_1}{N_c}\right)+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_1\left(12\ln\frac{m_b}{\mu}-2+f_I'+f_{II}'\right),$$
$$a_3' = \left(C_3+\frac{C_4}{N_c}\right)+\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_4\left(12\ln\frac{m_b}{\mu}-2+f_I'+f_{II}'\right), \tag{26}$$
$$a_5' = \left(C_5+\frac{C_6}{N_c}\right)-\frac{\alpha_s}{4\pi}\frac{C_F}{N_c}C_6\left(12\ln\frac{m_b}{\mu}+10+f_I'+f_{II}'\right).$$

Again, the vertex corrections associated with F_I are evaluated at renormalization scale $\mu \approx m_b$ and the spectator interactions associated with F_{II} are evaluated at $\sqrt{\Lambda_h m_b}$.

The functions f'_I and f'_{II} in eq. (26) have the following forms:

$$f'_I = \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}y \left[6 \ln\left[\left(x+\frac{y}{2}\right)\left(x+\frac{zy}{2}\right) \times \frac{y}{2}\left((z-1)x+\frac{zy}{2}\right)\right] + 4 - \frac{2x(1-z)}{x+\frac{zy}{2}} + \frac{zy-(1-z)x}{\frac{1}{2}\left((z-1)x+\frac{zy}{2}\right)}\right],$$
$$f'_{II} = \frac{8\pi^2}{N_c}\frac{f_K f_B}{m_B^2 F_1(m_{\psi'}^2)(1-z)} \times \int_0^1 \mathrm{d}\xi \frac{\phi_B(\xi)}{\xi}\int_0^1 \mathrm{d}y \frac{\phi_K(y)}{1-y}, \qquad (27)$$

where $z = m_{\psi'}^2/m_B^2$ and $F_1(m_{\psi'}^2) = 0.91$. One can easily get the functions in (27) by using the known results of $B \to J/\psi K$ in ref. [21], where J/ψ is described by LCDAs. We only need to replace the decay constant $f_{J/\psi}$ by $f_{2S} = \sqrt{\frac{3}{\pi m_{\psi'}}}R_{2S}(0)$ and choose the nonrelativistic limit form $\phi_{\mathrm{NR}}(u) = \delta(u-1/2)$ for LCDAs of J/ψ as in ref. [21].

According to eq. (1) we can write down the ratio of the decay rates directly:

$$R = \frac{\mathrm{Br}(B \to \psi'' K)}{\mathrm{Br}(B \to \psi' K)} = \left(\frac{1-r}{1-z}\right)\left|\frac{-i\mathcal{M}' \times \sin\theta + i\mathcal{M} \times \cos\theta}{i\mathcal{M}' \times \cos\theta + i\mathcal{M} \times \sin\theta}\right|^2. \qquad (28)$$

The ratio determined by experimental data is $R \approx 0.72$. Comparing it with our calculation, we can find that the mixing angle is $\theta = -26°$, or $\theta = +59°$. But the absolute branching ratio of $B \to \psi'' K$ is 5.9×10^{-5}, which is still about one order of magnitude lower than the experimental data in eq. (2).

4 Higher-twist effects and end-point singularities

In the last two sections, we have only considered the leading-twist spectator interactions. Generally, the contributions arising from higher-twist LCDAs of K meson will be suppressed by powers of $1/m_b$. However, as we have mentioned, the chirally enhanced scale $\mu_K \sim m_b$ in (12) ensures that the twist-3 contributions may be numerically large. It was discussed several years ago that these contributions may play important roles in the process of B meson to S-wave charmonia decays [22]. Here we consider the higher-twist effects in the D-wave charmonium production as well.

The distribution amplitude of the kaon to twist-3 have been given in (12), then we can find the twist-3 modifications to f_{II} and f'_{II} to be

$$f_{II}^3 = -\frac{16\pi^2}{N_c}\frac{f_K f_B}{m_B^2 F_1(m_{\psi''}^2)}\frac{r_K}{(1-r)^2}\int_0^1 \mathrm{d}\xi\frac{\phi_B(\xi)}{\xi} \times \int_0^1 \mathrm{d}y \frac{\phi_K^\sigma(y)}{6}\left(\frac{r}{(r-1)\,y^3}+\frac{1}{10\,y^2}\right),$$
$$f_{II}^{3'} = \frac{8\pi^2}{N_c}\frac{f_K f_B}{m_B^2 F_1(m_{\psi'}^2)}\frac{r_K}{(1-z)^2}\int_0^1 \mathrm{d}\xi\frac{\phi_B(\xi)}{\xi} \times \int_0^1 \mathrm{d}y \frac{\phi_K^\sigma(y)}{6}\frac{1}{y^2}, \qquad (29)$$

where $r_K = 2\mu_K/m_b$. Here, we can see that there exist logarithmic end-point singularities in both f_{II}^3 and $f_{II}^{3'}$. More seriously, there emerges a linear singularity in the function f_{II}^3 and we will parameterize it just like what we have done for the logarithmic ones:

$$\int_0^1 \frac{\mathrm{d}y}{y^2} = \frac{m_B}{\Lambda_h}. \qquad (30)$$

It is implicit that these singularities can be regularized by the gluon or light quark offshellness of order Λ_h^2 when we use (21) and (30). So when the offshellness is negative, the logarithmic singularity will receive large complex contributions from the implicit pole in the region of integration. They are common effects in soft rescattering processes. Then, following [25], we rewrite (21) as

$$X \equiv \int_0^1 \frac{\mathrm{d}y}{y} = \ln\left(\frac{m_B}{\Lambda_h}\right) + t, \qquad (31)$$

where t is a complex free parameter and we choose $|t|$ varying from 3 to 6 as suggested in [22]. Setting $|t| = 4.5$, $0 \le \delta \le \pi$, and the S-D mixing angle $\theta = -12°$, we can get the branching ratio curves of $B \to \psi(3770)K$ and $B \to \psi(3686)K$, which are shown in fig. 2.

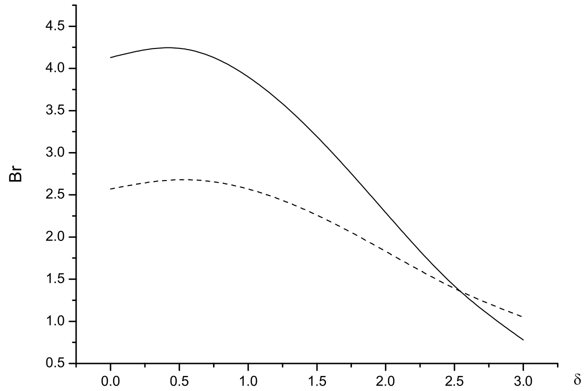

Fig. 2. Branching ratios of $B \to \psi(3770)K$ and $B \to \psi(3686)K$ (in units of 10^{-4}) as functions of δ. The dashed line is for $B \to \psi(3770)K$ and the solid line for $B \to \psi(3686)K$.

From fig. 2 we see that in the region with small δ the branching ratios are not very sensitive to the value of δ.

With a value of, say $\delta = \pi/8$, the branching ratios of $B \to \psi(3770)K$ and $B \to \psi(3686)K$ are found to be:

$$\begin{aligned}\mathrm{Br}(B^+ \to \psi'' K^+) &= 2.68 \times 10^{-4},\\ \mathrm{Br}(B^+ \to \psi' K^+) &= 4.25 \times 10^{-4}.\end{aligned} \tag{32}$$

From these values we can get $R = 0.63$, which fits the experimental data quite well. At the same time, the absolute branching ratios are both close to the experimental data. So we may conclude that when the higher-twist effects are taken into account and the S-D mixing is considered as well, the branching ratio of $B \to \psi(3770)K$ can become large enough to fit experimental data. If a smaller value for $|t|$ is used, the calculated decay rates are somewhat smaller, but still much more improved than the previous calculation. Here, the soft scattering effects in the spectator interactions have played an essential role.

5 Discussions and summary

In this paper, we study the $B^+ \to \psi(3770)K^+$ decay within the QCD factorization framework. If we treat $\psi(3770)$ as a pure 1^3D_1 state and use the leading-twist approximation for the kaon, we only get a very small branching ratio $\mathrm{Br}(B \to \psi'' K) = 1.13 \times 10^{-5}$, which is about 40 times lower than the experimental data.

We further introduce the S-D mixing, combined with the calculation for the $B^+ \to \psi(3686)K^+$ decay, but still use the leading-twist approximation for the kaon, then by fitting the observed ratio of $B^+ \to \psi(3770)K^+$ to $B^+ \to \psi(3686)K^+$, we find the required mixing angle to be about $\theta = -26°$ or $\theta = +59°$. These mixing angles are not consistent with that obtained from other experiments. Moreover, the absolute branching ratio of $B^+ \to \psi(3770)K^+$ is still one order of magnitude lower than the experimental data.

We then take the higher-twist effects into account. By choosing proper parameters to characterize the end-point singularities related to the soft spectator interactions, and taking the S-D mixing angle to be the widely accepted value $\theta = -12°$, we can get a much larger branching ratio, and it is then possible to make the calculated rate of $B^+ \to \psi(3770)K^+$ close to the data.

We would like to emphasize that in the present calculation it is the soft scattering effects in the spectator interactions that are essential in enhancing the decay rates, though there exist uncertainties for treating the soft singularities. Here, it might be useful to discuss the possible connection between the inclusive process $B \to \psi(3770) + \textit{anything}$ and the exclusive process $B \to \psi(3770)K$. In fact, with the nonrelativistic QCD (NRQCD) formalism [26] it was pointed out [27] (see also [28]) that for the D-wave charmonium inclusive production in B decays the color-octet $c\bar{c}$ operators in the weak-decay effective Hamiltonian may play the dominant role by producing a color-octet $c\bar{c}$ pair at short distances, which subsequently evolve to a color-singlet $c\bar{c}$ (the physical charmonium) by emitting soft gluons at long distances. When the emitted soft gluon interacts with and is absorbed by the spectator light quark, the process becomes an exclusive one, such as $B \to \psi(3770)K$ (the emitted soft gluons can of course hadronize into light hadrons without interactions with the spectator quark). If this picture makes sense, our observation in the present work that the soft scattering effects in the spectator interactions play the essential role in $B \to \psi(3770)K$ should be reasonable.

This may also be true for the B exclusive decays involving S-wave charmonia J/ψ [22,21] and η_c [29], where the calculations for $B \to J/\psi(\eta_c)K$ without twist-3 soft spectator contributions are much smaller than the observed rates, and the enhancement effect due to higher twist is emphasized in [22]. For the B exclusive decays involving P-wave charmonium states, the situation becomes even more puzzling, that is, the measured nonfactorizable $B \to \chi_{c0}K$ decay rate [30,31] is large, about an order of magnitude larger than that of other two nonfactorizable decays $B \to \chi_{c2}K$ [32] and $B \to h_cK$ [33]. These are not compatible with predictions based on the final-state rescattering model [34]. Some of these decays are also studied in the PQCD approach with k_t factorization [35], and in the light-cone sum rule approach [36]. In QCD factorization it is found that for the B exclusive decays involving P-wave charmonium states, there exist infrared divergences in the QCD vertex corrections [37,38]. However, if the twist-3 soft spectator interactions dominate, we might provide a possible explanation for the puzzle related to $B \to \chi_{c0}(\chi_{c2}, h_c)K$ decays, and this result will be presented elsewhere [39].

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